Population Uncertainty in Voluntary Contributions of Public Goods

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Abstract
I examine how the voluntary contribution of public goods is affected by uncertainty about the size of the relevant population. I show that when the number of players is uncertain and each player only knows the population distribution, the voluntary contribution level in a Nash equilibrium is higher than when the number of players is fixed at the mean of the population distribution, under mild assumptions. I also show that the voluntary contribution level decreases as the expected number of players increases, but the decreasing tendency is weaker than the prediction from the model with a certain population. My model can distinguish between voluntary contributions driven by warm-glow and by population uncertainty, so it provides a structural form to separate the warm-glow effect from the response to population uncertainty. The results from the lab experiments support many aspects of the theoretical predictions. The decreasing tendency is weak when the population size is random, and the lower bound of the population distribution mainly determines the subjects' decisions. When the population distribution is more volatile subjects contribute more. It is an unexpected observation that the salience of population uncertainty partly drive out warm-glow.

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1 Introduction

I examine how individuals contribute to the production of public goods when they do not know the exact number of participants in the contribution pool. Previous research has used lab experiments to determine what factors encourage individuals to contribute to provide public goods.\(^1\) The Voluntary Contributions Mechanism (VCM), or its variations, has been repeatedly revisited by verifying observations\(^2\), extending ideas\(^3\) or extrapolating results\(^4\) from the existing literature. One common result found in the vast majority of the previous studies is that some subjects seem to feel a ‘warm-glow’, which can be modeled by additional utility earned by the activity of giving itself (Cornes and Sandler (1984) and Andreoni (1989, 1990)). In terms of the experimental designs, another noticeable similarity of the previous research is that all experiment participants were certain how many other individuals make their decisions simultaneously, and accordingly certain how influential their contributions were.\(^5\) However, in many real world situations, a potential contributor does not know how many other contributors exist in the contribution pool: A voter does not know how many people with voting rights consider turning out or not, a charitable giver does not know how many others will consider donating for the relief of poor countries’ needy children, a voluntary watchman does not know how many neighbors can work for other time slots during the summer vacation period, and potluck party organizers do not know how many others will contribute since they do not know how many will consider coming. Does this uncertainty change individuals’ behavior? If so, how does it change?

To answer these questions, I model a voluntary contribution game with population uncertainty. A player who belongs to the game knows the distribution of population in the game, but does not know the exact number of players. This randomness is referred to as population uncertainty.

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\(^2\)Andreoni (1995a) found that, on average, about half of all cooperation comes from subjects who understand that non-cooperation maximizes their payoffs, but choose to cooperate out of some form of kindness. Brandts and Schram (2001) found that subjects’ behavior cannot be explained exclusively as the result of errors. Fischbacher et al. (2001) found in a one-shot public goods game that half of the subjects were conditional cooperators. Harbaugh and Krause (2000) conducted a public goods game with children and found older children’s behavior was similar to that of adults.


\(^5\)Uncertainty in population size has been ignored in the literature of voluntary contribution. Anecdotal evidence could be that the Z-Tree (Fischbacher (2007)), the most popular tool for designing various types of voluntary contribution experiments, treats the group size as a default parameter so it does not provide an option for including population uncertainty.
Though population uncertainty has been adopted in many other fields in applied microeconomics, to the best of my knowledge it has not been emphasized in the literature of voluntary contribution of public goods.

I show that when the number of players is random the voluntary contribution level in a symmetric Nash equilibrium is higher than when the number of players is fixed at the mean number of players when the marginal production of public goods is convex. The equilibrium voluntary contribution level decreases when the population size increases (Andreoni (2006)), and also when population uncertainty exists. I also show that the decreasing tendency of the equilibrium voluntary contribution level is weaker than the prediction from the model with a certain population when the population uncertainty increases as the mean population size increases. With these two findings, my model can distinguish between voluntary contributions driven by warm-glow and by the population uncertainty, so it provides a structural form to estimate the warm-glow effect and the response to the population uncertainty separately. The model contains a standard public goods provision model (Bergstrom et al. (1986)) as a special case, and also considers the warm-glow utility (Cornes and Sandler (1984), and Andreoni (1989, 1990)). Population uncertainty is described in a similar manner to that of Poisson games (Myerson (1998b), and Myerson and Wärneryd (2006)), but my model does not require having an unbounded support.

While theoretical predictions of population uncertainty in the voluntary contributions of public goods are clear, it is unclear how people respond to population uncertainty, as the self-interested agent models of voluntary contribution have been unsuccessful at predicting behavior. If people interpret the uncertain population size as another uncertainty or ‘risk’ of return on the contribution to the production of public goods, they might want to increase the allocation to the private goods consumption so that their utility can come from a more certain source. On the other hand, if risk-averse subjects worry more about the worse situation where the population size is realized to be low, or their risk aversion drives them to weight more on the lower realization of the population, they may want to contribute more. Another possibility is that the salience of population uncertainty may change a subject’s decision process because it implicitly encourages them to recognize the strategic aspects of the games. Under the null hypothesis of the self-interested rational agents, changes in their responses only depend on population uncertainty.

I conduct a series of experiments designed to test hypotheses about how population uncertainty affects voluntary contributions for public goods provision. The baseline experiment investigates whether and how people respond to population uncertainty in the VCM when production of public goods is linear in total contributions. The control group experiment investigates the effects of the group size with a decreasing convex marginal production. The treatment group experiment adds population uncertainty on the setup for the control group experiment. Subjects in the treatment group choose their contribution level given that they only know the population distribution (two different group sizes with equal probability), and their earnings are determined after the popu-

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6Since Myerson (1998b) introduced population uncertainty in games, studies in political economy actively use the idea of population uncertainty to understand the voter turnout. (Myerson (1998a), Piketty (2000), Dhillon and Peralta (2002), Bendor et al. (2003), Spenukh (2013)). In the context of contests, population uncertainty also played an important role. (Myerson and Wärneryd (2006), Münster (2006), Lim and Matros (2009))
tion size is realized. The main experiment combines both the control group and treatment group experiments for within-group comparisons. The experiments were conducted at the Cornell Lab for Experimental Economics & Decision Research (LEEDR) and employed undergraduate students at Cornell University. Except for including the randomness of the group size and the nonlinearity of the production function, the basic structure of the experiment resembled that of Palfrey and Prisbrey (1997). To minimize the effects of dynamic strategies I adopted random rematching (Andreoni and Croson (2008)). For the same reason, I did not tell subjects when the experiments would end.

The experimental results can be summarized as follows: (1) Salience of population uncertainty partly drives out warm-glow: In the linear VCM where their contributions can only be explained by warm-glow, contribution levels are halved. I call this a drive-out effect. (2) The decreasing tendency is weak under population uncertainty. The lower bound of the population distribution mainly determines the subjects’ decisions. (3) Once population uncertainty is salient, the more volatile population distribution brings more contributions. (4) After controlling for the drive-out effect, subjects respond to population uncertainty more than the theory predicts. These are new observations that cannot be captured by the previous experiments.

The rest of this paper is organized in the following way. Section 2 describes the model and section 3 shows the theoretical findings. Section 4 describes the design and procedure of the experiments and Section 5 highlights the experimental results. Section 6 concludes.

2 The Model

As a benchmark, consider a case where there are a fixed number of players. Let $N = \{1, 2, \ldots, n\}, n \geq 2$ denote a set of homogeneous potential contributors. Each contributor has endowment $y > 0$. There are one public good and one private good. Each contributor $i \in N$ contributes $g_i \geq 0$ to the supply of the public good and consumes a remainder $x_i = y - g_i$. The supply function of public good, $f : [0, y]^n \to \mathbb{R}_+$ is an increasing and strictly concave function of the sum of individual contributions. Let $G = f(\sum_{i=1}^n g_i)$ denote the supply of public good. The utility function for contributor $i$ is $U(x_i, g_i, G) = x_i + \psi(g_i) + v(G)$. The additively separable quasi-linear form is taken for simplicity, and relaxing this assumption does not change the directions of the main theoretical findings.

In a game with population uncertainty, the number of individual players is random. Formally, let $N_{1+} = \{2, 3, \ldots\}$ be the set of integers greater than 1, and $\pi : N_{1+} \to [0, 1]$ such that $\sum_{n=2}^{\infty} \pi(n) = 1$ be the common probability density function for the number of contributors. Let $\mu := \sum_{n=2}^{\infty} \pi(n) n$ denote the expected number of players. When $\pi$ is a degenerate function at $n$, it is reduced to the benchmark setup. I assume risk neutrality with respect to private consumption because in the context of the voluntary contribution games $x$ simply refers to the risk-neutral monetary value.

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7When the subjects are matched with the same group members for multiple periods, they have an incentive to behave strategically: If they can send credible signals for some rounds that they are cooperative, then they can leverage their earnings by betraying the belief and behaving non-cooperatively at the final round.

8The additively separable quasi-linear form is taken for simplicity, and relaxing this assumption does not change the directions of the main theoretical findings.
Now I consider this voluntary contribution game from the perspective of a single player.\(^9\) He knows that the population size will be realized from a distribution whose density function is \(\pi(n)\).\(^10\) This player’s objective is to choose the contribution level \(g\) to maximize his expected utility given indistinguishable others’ contributions denoted by \(\tilde{g}\).

**Definition 1.** A contribution level \(g^*\) is a symmetric equilibrium if

1. \(g^* = \arg \max_{g \in [0,y]} \sum_{n=2}^{\infty} \pi(n)\{y - g + \psi(g) + v(f(g + (n - 1)\tilde{g}))\}\), and
2. \(g^* = \tilde{g}\).

To exclude a trivial equilibrium where everyone contributes nothing, assume that some positive amount of the public good is desirable.

**Assumption 1.** \(\lim_{g \to 0} \psi'(g) + v'(f(g))f'(g) > 1\)

Assumption 1 states that even when no other players contribute, it is desirable to contribute non-zero amount of endowment to the production of public goods. Note that the right-hand side of the inequality is the marginal cost of increasing contribution in a quasi-linear utility function, so this assumption could be generalized to \(\lim_{g \to 0} U_2 + U_3 f' > \lim_{g \to 0} U_1\), where \(U_i\) is the partial derivative of \(U\) with respect to the \(i\)th argument. Assume further that \(y\) is sufficiently large enough for any two players not to contribute all endowments.

**Assumption 2.** \(\lim_{g \to y} \psi'(g) + v'(f(y + g))f'(y + g) < 1\)

Assumption 2 states that contributing all endowments is not desirable when another player contributes all endowments to the production of public goods.

The existence and the uniqueness of the symmetric voluntary contribution equilibrium are well-established in Cornes (2009).

### 3 Analysis

This section studies the simplest model where \(v(f) = f\). The player’s maximization problem simplifies to

\[
\max_{g_i \in [0,y]} \sum_{n=2}^{\infty} \pi(n)(y - g_i + \psi(g_i) + f(g + (n - 1)\tilde{g}))
\]

Due to Assumptions 1 and 2 corner solutions can be excluded, so the first order condition is

\[
\sum_{n=2}^{\infty} \pi(n) \left[\psi'(g^*) + f'(g^* + (n - 1)\tilde{g})\right] = 1.
\]  

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\(^9\)Since the population size is always greater than two in this model, we can assume there always exists a player and his/her match participating in this game. For slightly different setups about population uncertainty, see Myerson (1998b), and Myerson and Wärneryd (2006).

\(^10\)Some studies, especially in auction theories, deal with population uncertainty by introducing the pre-stage of the game. In the pre-stage, some of players are selected from the whole population pool and then they play a game. In this setup a single player should update the belief conditional on the fact that he is being selected because the fact implies that the population is more likely to be large. \(\pi(n)\) in this paper can be understood as a conditional belief that has already been updated.
Figure 1: An illustration of Proposition 1
This figure illustrates a special case of population uncertainty where the population is either 2 or $2\mu - 2$ with equal probability, and $\psi'(\cdot) = 0$. Since $f'(ng^c) > f'(ng^u)$, the individual contribution level under population uncertainty, $g^u$ is larger than the contribution level when population is certain, $g^c$. It requires that the marginal production is convex.

In the symmetric equilibrium where $g^* = \tilde{g}$, the first order condition becomes

$$\sum_{n=2}^{\infty} \pi(n)f'(ng^*) + \psi'(g^*) = 1. \quad (2)$$

Without population uncertainty, the first order condition is

$$f'(\mu g^*) + \psi'(g^*) = 1, \quad (3)$$

which is the same with the standard voluntary contribution model’s optimality condition shown by, for example, Bergstrom et al. (1986) if the warm-glow utility is constant or $\psi'(\cdot) = 0$. Since $\sum_{n=2}^{\infty} \pi(n) = 1$ and $\sum_{n=2}^{\infty} \pi(n)ng^* = \mu g^*$, we can read the equation (3) as $f'(\sum_{n=2}^{\infty} \pi(n)ng^*) + \psi'(g^*) = 1$. Then by Jensen’s inequality it immediately follows that the equilibrium contribution level with population uncertainty is greater than the level with certainty if the marginal production is convex. Figure 1 illustrates Proposition 1.

**Proposition 1.** Suppose $f'(\cdot)$ is convex. Let $g^u$ and $g^c$ denote the equilibrium contribution level with and without population uncertainty, respectively. Then $g^u \geq g^c$.

**Proof:** By Jensen’s inequality $\sum_{n=2}^{\infty} \pi(n)f'(ng^c) \geq f'(\sum_{n=2}^{\infty} \pi(n)ng^c) = 1 - \psi'(g^c)$. Since $f'(\cdot)$ is decreasing due to concavity, $g^u$ such that $\sum_{n=2}^{\infty} \pi(n)f'(ng^u) = 1 - \psi'(g^u)$ has to be greater than $g^c$. For the sake of contradiction, suppose $g^c > g^u$. Since $\psi'(\cdot)$ is increasing and concave, $\psi'(g^u) > \psi'(g^c)$, or $1 - \psi'(g^c) > 1 - \psi'(g^u)$. It implies $\sum_{n=2}^{\infty} \pi(n)f'(ng^c) > \sum_{n=2}^{\infty} \pi(n)f'(ng^u)$, and hence $g^u > g^c$, which contradicts the supposition. □
I claim that the convexity of marginal production is not demanding. Conventional production functions, such as \( \ln g \) and \( g^\alpha, \alpha \in (0, 1) \) meet the convexity of marginal production. Note that \( g^u \geq g^c \) holds with equality when the production function is linear, that is, the marginal production is constant.

The intuition to Proposition 1 is closely related to an observation which Myerson and Wärneryd (2006) found in contests: Under population uncertainty, the aggregate level of efforts (investments in the war of attrition, or biddings in the all-pay auction) is smaller than when the population is certain. Both in contests for private prizes and voluntary contributions to public goods, each individual’s marginal cost of investment is certainly known to her/him. The population uncertainty plays a role in the marginal benefit of investment. In contests where payoffs are only given to winners, players are reluctant to exert additional effort because their marginal benefit would be small when the population is realized to be large. In voluntary contribution games where payoffs are distributed to all participants, players are encouraged by the possibility of small population which will make their contributions the most influential.

Another direct observation from Proposition 1 is that the mean-preserving spread of a population distribution leads a larger contribution in equilibrium.

**Corollary 1.** Suppose \( f'() \) is convex. Let \( \Pi_1 \) denote the population distribution and \( \Pi_2 \) denote the mean-preserving spread of \( \Pi_1 \). Let \( g_1^u \) and \( g_2^u \) denote the equilibrium contribution level with the population uncertainty whose distribution follows \( \Pi_1 \) and \( \Pi_2 \), respectively. Then \( g_2^u \geq g_1^u \).

**Proof:** Let \( \pi_1(n) \) and \( \pi_2(n) \) denotes the probability density functions of \( \Pi_1 \) and \( \Pi_2 \), respectively. Applying Jensen’s inequality again, we have \( \sum_{n=2}^{\infty} \pi_2(n)g_1^u(n) > \sum_{n=2}^{\infty} \pi_1(n)f'(ng_1^u) \). The remaining proof is the same as that of Proposition 1. \( \square \)

If the direct utility from the contribution is additively separable to the utility of the public goods provided, Proposition 1 holds regardless of the existence of the warm-glow utility. Thus it suggests that individuals’ pro-social behavior may be decomposed by warm-glow and the player’s response to population uncertainty. The following proposition helps to identify those two factors. To emphasize the expected number of population, let \( g^u(n) \) and \( g^c(n) \) denote the symmetric equilibrium contribution level with and without population uncertainty when the expected number of population is \( n \). When population uncertainty grows, the free riding incentive grows more slowly. For describing a limiting behavior, I use the big \( O \) notation.

**Proposition 2.** Suppose \( U(x_i, g_i, G) = x_i + \alpha \ln g_i + b \ln G \). If the number of players is randomly drawn from a discrete uniform distribution \( U[2, 2n - 2] \), then \( g^c(n) = O(1/n) \) and \( g^u(n) \approx O(\ln(2n)/n) \).

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11Admittedly, not every increasing concave function has a positive third order derivative. For example, \( f(g) = \alpha g - g^2, \alpha > 0 \) is concave on \( \mathbb{R} \) and \( f'(g) = \alpha - 2g \) is not convex. However such functions are in general increasing only with an upper bound. In the example, the production function is decreasing when \( g \) is greater than \( \alpha/2 \).

12Andreoni (2006) used the impure altruism as a synonym of warm-glow. Here the warm-glow is referred to the direct utility from the giving, captured by \( \psi() \).

13Let \( f(x) \) and \( g(x) \) be two functions on \( \mathbb{R} \). A function \( f(x) \) is \( O(g(x)) \) as \( x \to \infty \), or \( f(x) \) is the same-order relation with \( g(x) \) if there exist positive constants \( M \) and \( X \) such that \( |f(x)/g(x)| \leq M \), for all \( x \geq X \).
Proof: To avoid unnecessary algebra, consider instead that the number is drawn from a discrete uniform distribution $U[1, 2n - 1]$. This modification does not change the limiting behavior with respect to the convergence speed. From equation (3), $g^a(n) = \alpha + b/n$, which is of $1/n$ order. From equation (2), $g^u(n) = \alpha + b/n \sum_{i=1}^{2n-1} 1/i$. Since $\sum_{i=1}^{2n-1} 1/i$ is a left Riemann sum of $1/x$ from 1 to $2n$ and $1/x$ is decreasing in $x$, $\sum_{i=1}^{2n-1} 1/i > \int_1^{2n} 1/x dx = \ln(2n)$. Similarly, $\sum_{i=1}^{2n-1} 1/(i+1)$ is a right Riemann sum of $1/x$ from 1 to $2n$, $\sum_{i=1}^{2n-1} 1/(i+1) < \ln(2n)$, or $\sum_{i=1}^{2n-1} 1/i < \ln(2n) + 1 - 1/(2n)$. Thus $\alpha + b/n \ln(2n) < g^u(n) < \alpha + b/n (\ln(2n) + 1 - 1/(2n))$, and $b/n - b/(2n^2)$ shrinks to zero faster than $b/n \ln(2n)$ as $n$ goes to infinity. $g^u(n)$ approaches to $\alpha + b/n \ln(2n)$, which is of $\ln(2n)/n$ order. \( \square \)

Proposition 2 states that as $n$, the mean population size goes to infinity, both $g^r(n)$ and $g^u(n)$ shrink to $\alpha$, that is, the voluntary contribution level asymptotically depends on warm-glow only (Andreoni (2006)). However, the convergence speed to $\alpha$ is $\ln(2n)$ times slower when population uncertainty grows as the mean population size grows.\(^{14}\) A general idea of Proposition 2 can be applied to many similar cases where individual’s observed behavior is believed to be a combination of a decision-theoretic behavior (here, $\alpha$) and a strategic behavior (here, $b/n$): Population uncertainty mainly affects the strategic behavior so it provides a structural way of decomposing the decision-theoretic behavior from the strategic behavior, as long as a social planner (or an experimenter in lab experiments) can perturb population uncertainty.

One practical way of utilizing Proposition 1 and Proposition 2 to laboratory experiments is to compare subjects’ decision when the group size is $\mu_k$, $k = 1, 2, \ldots$, to when the size is either $\mu_k - d_k$ or $\mu_k + d_k$ with 50% of chance each, where $\mu_{k+1} < \mu_k$ and $d_{k+1} > d_k$. If subjects behave as the theory predicts, then $g^u(\mu_k) > g^r(\mu_k)$ and $|\Delta g^u(\mu_k)/\Delta \mu_k| < |\Delta g^r(\mu_k)/\Delta \mu_k|$ for all $k$. Though practically unlikely, if the setup for the laboratory experiments is exactly the same as that in Proposition 2, one may test a following nonlinear regression model and hypothesis.

\[
\begin{align*}
g_i &= \alpha + b \frac{\ln(2n)}{n} + \epsilon_i, \quad E(\epsilon_i) = 0 \\
H_0 : \alpha &= 0
\end{align*}
\]  

, where potential contributors have no warm-glow ($\alpha = 0$) under the null hypothesis. $b$ is specified by a production function for the laboratory experiments.

While it is theoretically clear how people should respond to population uncertainty with the nonlinear production function of public goods in equilibrium, it is unclear how subjects are actually responding to population uncertainty in the laboratory experiments. There may be two contrasting directions in the response to population uncertainty. If experiment participants interpret the uncertain population size as another uncertainty or ‘risk’ of return on the contribution to the production of public goods, they might want to increase the allocation to their private account so that their

\(^{14}\)This convergence speed depends on the functional form of utility, and the shape of the population distribution. The upshot of this observation is that if population uncertainty gets larger as the expected number of population grows, players are more reluctant to free-ride.
utility can come from a more certain source. On the other hand, if conservative participants worry more about the worse situation where the population size is realized to be low, that is, their risk averseness drives them to put more weight on the lower realization of the population, they may want to contribute more. In this case, population uncertainty may still play a role even when the mean population size is large if a potential contributor subjectively perceives that the probability of a small population is substantially high. On top of these contrasting directions, another possibility is that the salience of population uncertainty may change the subjects’ decision process by directly affecting their warm-glow. If population uncertainty makes the subjects consider the strategic aspects of the experiment more it may encourage them to behave more rationally and self-interested, that is, population uncertainty may drive out warm-glow. Yet we cannot exclude a possibility that population uncertainty encourages them to be more altruistic. All of these contrasting directions can be captured in the following testable nonlinear regression model and hypothesis,

\[ g_i = \alpha - \gamma 1_U + b \ln\left(\frac{2(n - \delta)}{n - \delta}\right) + \varepsilon_i, \quad E(\varepsilon_i) = 0 \]

\[ H_0 : \alpha = 0, \gamma = 0, \delta = 0 \]

(5)

where \( 1_U \) is an indicator function of population uncertainty, \( \gamma \) denotes the driving-out effect of population uncertainty on warm-glow, and \( \delta \) denotes the subjective concern for a lower realization of population. Under the null hypothesis, potential contributors are self-interested (\( \alpha = 0 \)), population uncertainty does not alter their warm-glow, if any, (\( \gamma = 0 \)), and they rationally respond to population uncertainty by accepting the population distribution objectively (\( \delta = 0 \)). A positive \( \alpha \) implies that people have the warm-glow utility. When \( \alpha \) is positive, a positive \( \gamma \) implies that population uncertainty drives out warm-glow. Meanwhile, a negative \( \gamma \) suggests that population uncertainty makes subjects more altruistic. If \( \delta \) is positive then it means that people worry more for the lower realization of population, or increase their voluntary contribution level to respond to population uncertainty. A negative \( \delta \) means that people behave as if they have a higher realization of population so decrease their voluntary contribution level. A sufficiently large negative \( \delta \) can imply that people would rather choose a contribution level simply by their warm-glow. A sufficiently large positive \( \delta \) can imply that even when there is sufficiently large population interested in a public good provision, still they contribute more when the population size is uncertain.

The experimental design is described in the following section. The experiments have a simpler form than the setup for Proposition 2 due to lab capacity and other practical restrictions, but the fundamental ideas that I try to capture are the same.

\( ^{15} \)This argument is in line with the “selection principle” of risk-dominance in the coordination game (Harsanyi and Selten (1988).) The underlying idea of risk-dominance equilibrium is that economic agents are unlikely to choose a payoff-dominating equilibrium strategy if such an equilibrium strategy requires many other agents’ cooperation or similar action. Though the equilibrium in this voluntary contribution game is unique, participants may dislike a situation where many others should behave as expected.
4 Experimental Design and Procedures

To test how much of a voluntary contribution is driven by warm-glow and by the response to population uncertainty, I conduct a series of experiments with four different groups, which are summarized in Table 1.

The purpose of the baseline experiments is to check if warm-glow is affected by population uncertainty. The main experiments are to examine how subjects respond to population uncertainty, the control experiments are to check if warm-glow is affected by the group size with a nonlinear production function, and the treatment experiments are to observe how subjects respond to the volatility of population uncertainty. Details follow.

Subjects in the baseline experiments play a series of linear voluntary contribution games: They are endowed with 10 tokens per round, and their task is to allocate the tokens to a private account and a group account in order to earn as much as they could. Tokens allocated to the private account earn 10 cents per token. Tokens allocated to the group account earn 3 cents per token for each member of the group. For example, if one participant allocates eight tokens in the private account and his group gathers 11 tokens in the group account, then he earns 113 cents (10*8 + 3*11) for the round. As long as the group size is greater than or equal to four, the Pareto-optimal contribution to the group account is 10, while the non-cooperative Nash equilibrium contribution is zero. With this linear production function of public goods, population uncertainty by itself plays no role as long as the subjects’ warm-glow is unaffected by population uncertainty. Baseline experiments test whether the salience of population uncertainty directly affects warm-glow. Subjects make decisions under four different predetermined situations (or two ‘control’ module experiments and two ‘treatment’ module experiments) in terms of the group size or the uncertainty of the size: In the control module experiment they make decisions when the group size is fixed at \( n_B \in \{4, 8\} \), where \( n_B \) is randomly drawn at the beginning of each round. In the treatment module experiment, they are informed at the beginning of each round that the group size will be either \( n_1^B \) or \( n_2^B \) with a 50% chance of each, where \((n_1^B, n_2^B) \in \{(4, 8), (4, 12)\})\). The actual group size is realized after they make a choice. They practice many times with virtual participants and understand what the game is about and how their strategy should be before the actual games start. The group

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16 In general, let \( y, n, p \) and \( k \) denote the endowment per round, the group size, the earnings per token in the private account and the earnings per token in the group account, respectively. If \( k < p < nk \), then the Nash equilibrium contribution level is zero because the marginal benefit of contribution, \( k \), is smaller than the marginal cost of contribution, \( p \), while the Pareto-optimal contribution level is \( y \) because the sum of the marginal benefit, \( nk \), is greater than the marginal cost of contribution.

17 Pseudo-codes for the determination of the group sizes are provided for clarification as follows:

```
module1list=[4,8]; module2list=[[4,8], [4,12]]; round1=random.choose(module1list,1); round2uncertainty=random.choose(module2list,1); round2realized=random.choose(round2uncertainty,1).
```

18 Subjects are allowed ten minutes to play with tutorials. The tutorials consist of a detailed introduction and a series of games with artificial experimental participants that form a group. They are told that the computers make their decisions randomly, thus their decisions may or may not be similar to those of real participants. In actual experiments, ten minutes were sufficient for most participants to play more than ten rounds of the game. On average they practiced with the tutorial for 16 rounds. See Appendix B for screen captures of the tutorial.

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10
size or the uncertainty of the size is randomly drawn from the predetermined list. Some studies including Andreoni and Croson (2008) consider the unintended possibility of subjects’ strategic behavior when the group members are unchanged for all rounds: Once the groups are formed in the beginning, for the first several rounds a member may send a false signal to other members so that the others might believe they are altruistic, and then they could exploit this belief in the final rounds. To exclude this strategic behavior every group is reformed every round. The experiments run for 20 rounds, 10 rounds for the control module and another 10 for the treatment. To minimize a final-round bias, subjects are uninformed about when the final round is.

The main experiments are similar to the baseline experiments. The only difference is that subjects earn money from tokens assigned to the group account according to Table 2, which represents a production function of $30 \ln(G)$.\footnote{During pilot trials, experiment participants understood the production schedule better with the table rather than either with an exact, but far longer conversion table or with a calculator and formula.} Since inequalities in Proposition 1 and Corollary 1 hold strictly when the marginal production is convex, such a nonlinear production function is necessary to see the effect of population uncertainty. Based on the total number of tokens allocated to the group account, every participant in the group earns the corresponding amount of money in the next column, whether or not she has contributed to the group account. For example, if 11 tokens are collected in the group account, no matter who contributes how much, every participant in the group earns 73 cents for the round. If a subject whose group gathers 11 tokens in the group account allocates eight tokens in her private account, she earns 153 cents ($73 + 10 \times 8$) for the round. If a group collects zero tokens in the group account, every subject in such a group forfeits 10 cents.\footnote{In order for the production function to exactly represent $30 \ln(G)$, every earning should be forfeited when $G = 0$ since $\ln(0) = -\infty$. I admit that it could have brought more significant differences in responses.}

The control group experiments are conducted in two sessions and another two sessions are for the treatment group experiments. Each session runs for 20 rounds. In the control experiment they are informed that the group size is fixed at $n^C \in \{4,6,8,12\}$ at the beginning of each round, where $n^C$ is randomly drawn per round. The Nash equilibrium for this case is $30/(10n^C) = 3/n^C$. In the treatment experiment they are informed that the group size would be either $n^T_1$ or $n^T_2$, where $(n^T_1, n^T_2) = \{(2,6), (4,8), (2,12), (4,12)\}$. The Nash equilibrium for this case is $30/10 \left(1/(2n^T_1) + 1/(2n^T_2)\right) = 3/2(1/n^T_1 + 1/n^T_2)$. For within-group comparisons, in another two sessions subjects do both control and treatment module experiments for 10 rounds each. Subjects are informed that the group size is fixed at $n^M \in \{4,6,8\}$ in the control module and the group size will be either $n^M_1$ or $n^M_2$, where $(n^M_1, n^M_2) \in \{(2,6), (4,8), (4,12)\}$. They also practice sufficiently with virtual participants, and in the real games the group sizes and uncertainties come in a random order. By setting the order of the rounds randomly, the experimenter’s intention plays a minimal role.

All experimental sessions were conducted at the Laboratory for Experimental Economics & Decision Research (LEEDR) at Cornell University from June 9 to June 20, 2014. Python and its application Pygame were used to computerize all games. Experiments were advertised in summer
classes, and indicated the anticipated average earnings for participating in the experiment would be $34, including a participation reward of $5. Participants were Cornell undergraduate students and were recruited through the Laboratory’s online recruitment system. Sessions ran from 60 minutes to 80 minutes depending on treatment specifications. Due to no-show subjects, the session sizes varied from 15 to 24,\textsuperscript{21} and a total of 141 subjects participated in the experiments. Participants were randomly assigned to separate desks equipped with a computer interface. They were not allowed to communicate with other participants during the experiment. It was also emphasized to participants that their allocation decisions would be anonymous. They read instructions and practiced with a tutorial program. An instructor answered all questions until every participant thoroughly understood the experiment.

Participants earned $33.71 on average, which is close to the advertised earnings expectation of $34. Only one participant allocated all of his tokens to the group account for all rounds and earned the lowest amount, $16.28, and the rest earned more than $22. The highest amount earned was $43.99. No group collected zero tokens so no one’s endowment was forfeited.

\section{Results}

The main results from the lab experiments are summarized as follows:

1. The salience of population uncertainty partly drives out warm-glow. In the linear VCM where their contributions can only be explained by warm-glow, contribution levels are halved.

2. The decreasing tendency is weak under population uncertainty. Indeed, the lower bound of the population distribution mainly determines the subjects' decisions.

3. The more volatile the population, the more subjects contribute.

4. After controlling for the driving-out effects, subjects respond to the population uncertainty more than the theory predicts.

Most of those results are completely new observations which cannot be captured using the setups for previous experiments.

\subsection{Warm-glow is partly driven out.}

Subjects who gave more than zero in the linear public goods production environment contributed less when there was population uncertainty. Though this suggests that it might be bad to keep population uncertainty to increase individual’s voluntary contribution level so it may deteriorate my theoretical predictions in the main experiments, I report this first because this is neither observed by any previous studies nor theoretically predicted.

\textsuperscript{21}The lab has 24 computers for participants, and accordingly the experiment was designed by assuming all 24 signed-up participants would attend. The divisors of 24 were specifically chosen as group sizes. Since I could not divide 15 into any group size that I designed, I created some ‘dummy players’ who chose the average decision for the round to substitute for absences.
The results from the baseline experiments are summarized in Table 3. In the baseline experiments, where the production function is linear, there is no theoretical difference between the equilibrium contributions with and without population uncertainty. As long as the group size is greater than four the Nash equilibrium contribution level is zero. Among 44 subjects who participated in the baseline experiments 45.45% of subjects (20 subjects) kept contributing nothing as the theory predicts, or contributed nothing except for one or two rounds of small contributions. Seven subjects (15.91%) maintained a consistent contribution level with little variation (a small change for only one or two rounds). An interesting observation is from the remaining 36.36% of subjects who kept contributing a nonzero amount of tokens to the group account, that is, showed warm-glow. Most of them contributed less with population uncertainty. On average the contribution level significantly decreased by 0.5361 with a mean group size of six and by 0.6687 tokens with a mean group size of eight. Since all subjects contributed on average 1.6591 tokens to the group account when there is no population uncertainty, their warm-glow is decreased by more than 32% when there was population uncertainty. When comparing the average contribution among those who contributed such a decrease becomes more distinct. Their contribution level was halved with population uncertainty, from 3.6077 to 1.8385. In one session the treatment module was conducted first, but this change of the order brought no significant differences. The learning effect was insignificant.

This result was interesting and it had never been observed in the previous studies, and there are no theories behind this observation: If they were to be risk averse, then they should not have contributed, even without population uncertainty. If they were to recognize population uncertainty as another ‘risk’, reducing their contribution does not help to decrease such a risk. Even if their rationality was bounded as in the level-k theories (Camerer et al. (2004) and Costa-Gomes and Crawford (2006)), their optimal decision should be to contribute zero as long as $k \geq 1$. If those who contributed were the $L_0$, that is, if such players made choices randomly, such significant differences should not be expected.

One may explain that the salience of population uncertainty encouraged subjects to recognize more of the strategic aspects of the game situation. Many studies found that the social preferences are affected by economic incentives (Bowles and Polania-Reyes (2012)), peers (Gächter et al. (2013)), risks (Saito (2013)) and/or contexts (Keisner et al. (2013)), so it may be plausible to interpret that subjects’ preferences were affected. Though it has been noted that the salience of uncertainty made people to consider fairness more (van den Bos (2001)), the experimental/empirical evidence has been meager as to how social preferences are affected by the salience of uncertainty.

The change in warm-glow was not due to the changes in the group size. The results from the control group experiment are summarized in Table 4. Without population uncertainty the average

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I intended to compare a fixed group size (for example, 8) with a random group size with the same mean (for example, 4 or 12 with 50% chance of each). Some claimed that I have to compare the weighted average of contributions from two fixed group sizes (for example, 4 and 8) and its corresponding average contributions (for example, 4 or 8). I adopted both comparison methods. The differences were statistically significant at the one percent significance level.
contribution levels tended to decrease as theory predicts and many previous studies have verified (Andreoni (2006)). When the group size was four, the average contribution level was 2.4048. It decreased to 2.3333, 2.0595 and 2.0340 for groups of six, eight and twelve, respectively. The last column of Table 4 shows the difference between the actual contribution levels with the equilibrium contribution levels. Interpreting such differences as their warm-glow, there is no significant pattern that the size of the group affects warm-glow. The contribution level due to warm-glow stayed around 1.6548 to 1.8333 tokens and the differences are not statistically significant. Another observation is that such contribution levels due to warm-glow in the control group experiment were almost identical to 1.6591, the average contribution level of the baseline experiments without population uncertainty.

[TABLE 4 AROUND HERE]

5.2 The lower bound of population distribution mainly determines the contribution levels.

With population uncertainty the average contribution level did not decrease with the mean group size. Rather, the average contribution level was larger with the smaller lower bound of the population distribution. The results from the treatment group experiments are summarized in Table 5.

[TABLE 5 AROUND HERE]

When the size of the group is either four or twelve, that is, when the mean group size is eight, the average contribution level was 2.0242. However, when the size of the group is two or twelve, a mean size of seven, the average contribution level was 2.5238. A similar tendency was observed with the smaller group sizes. When the size of the group was either two or six, the average contribution level was 2.2239, while when the size of the group was either four or eight, the average was 1.9515. In short, the contribution level was mainly determined by the lower bound of the population distribution. When the lower bound was two, subjects contributed more, regardless of the mean group size. Note that the fundamental idea of Proposition (1) is from the individuals’ responses to the possibility of realization of a lower population size, so this observation is as implied. Also, though it should be further verified with a larger population size, this observation may suggest that an individual’s voluntary contribution behaviors are affected by their subjective concern for the lower realization of the contribution pool population. In other words, the potential ‘threat’ of a small contribution pool encourages individuals to contribute to the production of public goods.

Unlike the control group experiment, the difference between the actual average contribution level and the equilibrium contribution level cannot be understood as warm-glow, as we observed that population uncertainty partly drives out warm-glow. The overall average contribution levels are not significantly different to those in the control group experiment.
5.3 The more volatile the population distribution, the more people contribute.

As predicted in Corollary (1), subjects contributed more when the population distribution was more volatile but with almost the same mean. In the treatment group experiments the difference between 2.5238, the average contribution when the group size was either two or twelve, and 1.9515 when it was either four or eight, is statistically significant at the one percent significance level.\(^{23}\) Though statistically insignificant, a similar pattern was observed in the main experiments. In the main experiments, the average contribution level when the group size was either four or eight, 1.7125, was smaller than 1.7902, the average contribution when the size was either two or twelve. It was impossible to solely extract the effect of the variance of the population distribution because with population uncertainty subjects behaved in a more self-interested manner, as observed in the baseline experiments.

Institutions and organizations that rely heavily on voluntary contributions cannot change the size of the contribution pool in the short term. Still, this observation has practical value in that it suggests the size of the contribution pool should not be mentioned so as to increase the voluntary contribution level.

5.4 Subjects respond to population uncertainty more than the theory predicts.

Table 6 summarizes the results from the main experiments. Results from the control module and the treatment module resembled those from the control group experiment and the treatment group experiments: In the control module, the average contribution level decreased as the group size increased, but after subtracting the equilibrium contribution level their warm-glow was not affected by the group size. In the treatment module, the average contribution level was higher when the lower bound of the population distribution was two. As predicted in Proposition(1), the contribution levels in the treatment module were higher than those in the corresponding control module. Although positive, all differences are not statistically significant. This insignificance was mainly due to the opposite effect of population uncertainty that partly drives out warm-glow as observed in the baseline experiment.

[TABLE 6 AROUND HERE]

The last column of Table 6 shows the driving-out-effect-adjusted contribution level. In the baseline experiments the average contribution level with population uncertainty was at least 0.5361 tokens smaller than that without population uncertainty. After controlling for the Nash equilibrium contribution level I adjusted the net contribution level by adding 0.5361. With this adjustment, the differences between the contribution levels with and without population uncertainty are more

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\(^{23}\)Note the mean group size is slightly different. When the group size is either four or eight the mean is six, while it is seven when the size is either two or twelve. Since the lab capacity is 24, I chose the group sizes that are divisible into 24. Given this restriction, ‘4 or 8’ and ‘2 or 12’ are the best possible sets to compare.
distinctive. The difference between 1.4175 (the adjusted average contribution level when the group size is either two or six) and 1.0518 (the contribution level when the group size is four) was statistically significant at the five percent significance level. The difference between 1.6861 (when the group size was four or eight) and 1.0952 (when the size was six) was also significant at the one percent significance level. The other case also had a positive difference, but it was insignificant.

6 Conclusions

Population uncertainty has been rarely studied in the literature of voluntary contribution of public goods despite its importance. With population uncertainty the equilibrium voluntary contribution level is greater than the equilibrium contribution level when the population size is fixed at the mean of the population distribution. As the population size grows the equilibrium contribution level decreases, but the decreasing tendency is weaker than the prediction from the model with a certain population. From the lab experiments we learned that an individual's voluntary contribution level does respond to population uncertainty and could verify many aspects of the theoretical predictions. Subjects were more concerned with a lower realization from the population distribution. When the population distribution was more volatile, subjects contributed more. These observations suggest that the institutions and organizations that rely heavily on individual voluntary contributions may bring in more contributions by suppressing the exact size of the population.

The salience of population uncertainty drove out warm-glow. This is an unexpected, but interesting, observation that previous studies could not capture. It is puzzling because once population uncertainty is salient, the more volatile population distribution brings more contributions. Many behavioral studies have considered many forms of certainty effects, but the certainty in population size is distinct because it appeals to individuals' strategic aspects while other forms of certainties appeal to an individual's risk preferences. I interpreted that such changes make people recognize the strategic aspects of the game situation more, but admit that it may have other interpretations.

There are many directions to extend this study. Many previous studies on the Voluntary Contributions Mechanism have considered the effects of individual characteristics such as gender, education and risk preference. Those studies could be redone by adding population uncertainty to answer how individual characteristics bring different responses to population uncertainty. Also, although this study has some theoretical predictions for a large population, it still is unclear whether the actual contribution levels would increase or decrease with population uncertainty. Cooperation with charities to have field experiments would be ideal because charitable giving is the contribution that brings little practical benefits to the donors. If population uncertainty drives out the pure altruistic aspects of charitable giving the contribution level would decrease.

Tables
Table 1: Summary of Experimental Design

<table>
<thead>
<tr>
<th>N per session ≤ 24</th>
<th>The size of each group</th>
<th>#Sessions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline ((f(G) = 3 \sum g_i))</td>
<td>(n = 4, 8, 4or8 or 4or12)</td>
<td>2</td>
</tr>
<tr>
<td>Control ((f(G) = 30 \ln(\sum g_i)))</td>
<td>(n = 4, 6, 8 or 12)</td>
<td>1*</td>
</tr>
<tr>
<td>Treatment ((f(G) = 30 \ln(\sum g_i)))</td>
<td>(n = 2or6, 4or8, 2or12 or 4or12)</td>
<td>2</td>
</tr>
<tr>
<td>Main (Control + Treatment)</td>
<td>(n = 2 or 6, 4 or 8, 2 or 12)</td>
<td>2</td>
</tr>
</tbody>
</table>

* In each round, 10 tokens are given. Experiments last 20 rounds.

* Subjects’ task: Allocate the tokens to a private account and a group account.

Baseline experiments are to check if warm-glow is affected by population uncertainty. With a linear production function of the public good, population uncertainty should not play a role as long as the warm-glow utility is additively separable. Main experiments are to examine how subjects respond to population uncertainty. \(n = 2or6\) denotes that the size of the group will be either 2 or 6 with equal probability, and the actual group size will be realized after subjects made their decisions.

* Due to technical glitches, data from one session for the control group experiment was not collected.

Table 2: Earnings from the group account

<table>
<thead>
<tr>
<th>Total tokens</th>
<th>earning(¢)</th>
<th>Total tokens</th>
<th>earning(¢)</th>
<th>Total tokens</th>
<th>earning(¢)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-10</td>
<td>8</td>
<td>62</td>
<td>30~39</td>
<td>106</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>9</td>
<td>66</td>
<td>40~49</td>
<td>114</td>
</tr>
<tr>
<td>2</td>
<td>21</td>
<td>10</td>
<td>69</td>
<td>50~59</td>
<td>120</td>
</tr>
<tr>
<td>3</td>
<td>33</td>
<td>11~12</td>
<td>73</td>
<td>60~69</td>
<td>125</td>
</tr>
<tr>
<td>4</td>
<td>42</td>
<td>13~15</td>
<td>79</td>
<td>70~79</td>
<td>129</td>
</tr>
<tr>
<td>5</td>
<td>48</td>
<td>16~19</td>
<td>86</td>
<td>80~89</td>
<td>133</td>
</tr>
<tr>
<td>6</td>
<td>54</td>
<td>20~24</td>
<td>93</td>
<td>90~99</td>
<td>136</td>
</tr>
<tr>
<td>7</td>
<td>58</td>
<td>25~29</td>
<td>99</td>
<td>100~109</td>
<td>139</td>
</tr>
</tbody>
</table>

This table maps the total number of tokens gathered in the group account to the total value of those tokens. For example, if subjects in a group together allocate 11 tokens to the group account, then every member in the group will earn €73. If every subject in a group allocates zero tokens to the group account, they are forfeited by €10.
Table 3: Baseline Experiments

<table>
<thead>
<tr>
<th>Module</th>
<th>Obs.</th>
<th>Group Size</th>
<th>Mean</th>
<th>NE</th>
<th>St.Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>'control'</td>
<td>224</td>
<td>4</td>
<td>1.2366</td>
<td>0</td>
<td>2.3718</td>
</tr>
<tr>
<td></td>
<td>216</td>
<td>8</td>
<td>2.0972</td>
<td>0</td>
<td>2.9531</td>
</tr>
<tr>
<td>'treatment'</td>
<td>244</td>
<td>4 or 8</td>
<td>1.1230</td>
<td>0</td>
<td>2.1491</td>
</tr>
<tr>
<td></td>
<td>196</td>
<td>4 or 12</td>
<td>1.4286</td>
<td>0</td>
<td>2.1460</td>
</tr>
</tbody>
</table>

* Difference between '4 and 8' and '4 or 8': 0.5361***

The fourth column and the sixth column show the average contribution levels and the standard deviations, respectively. The fifth column shows the Nash equilibrium contribution level. Total 44 subjects (20 in the first session, 24 in the second session) participated in the baseline experiments. In the ‘control’ module they played a standard VCM. In the ‘treatment’ module their task was the same, but the size of the group is realized after they made a decision. I compared a weighted average contribution level when the group size was fixed at four and at eight (in the table it is denoted by ‘4 and 8’) with an average contribution level when the group size was either four or eight with equal chance (‘4 or 8’). The difference between these two is 0.5361 tokens and its p-value is 0.0085. I also compared a fixed group size eight (‘8’) with a random group size with four or twelve (‘4 or 12’). The difference between these two is 0.6687 tokens and its p-value is 0.0047. Both differences were statistically significant with the 1 percent significance level.

Table 4: Control Group Experiments

<table>
<thead>
<tr>
<th>Group Size</th>
<th>Obs.</th>
<th>Mean</th>
<th>NE</th>
<th>St.Dev.</th>
<th>Warm-glow(*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>126</td>
<td>2.4048</td>
<td>0.7500</td>
<td>1.7672</td>
<td>1.6548</td>
</tr>
<tr>
<td>6</td>
<td>63</td>
<td>2.3333</td>
<td>0.5000</td>
<td>1.9757</td>
<td>1.8333</td>
</tr>
<tr>
<td>8</td>
<td>84</td>
<td>2.0595</td>
<td>0.3750</td>
<td>1.7518</td>
<td>1.6845</td>
</tr>
<tr>
<td>12</td>
<td>147</td>
<td>2.0340</td>
<td>0.2500</td>
<td>1.8335</td>
<td>1.7840</td>
</tr>
</tbody>
</table>

Total 21 subjects participated in the Control Group experiment. In each round the size of the group is randomly determined and displayed on the top of the game screen. Difference numbers of observations are due to such randomness. The average contribution level decreases as the size of the group increases, as the theory predicts. However, warm-glow stayed from 1.6548 to 1.8333 tokens, which is almost identical to 1.6591, the average contribution level of the baseline experiments without population uncertainty.

* The difference between the average contribution level and the Nash equilibrium contribution level is used as a proxy measure of the warm-glow utility.
**Table 5: Treatment Group Experiments**

<table>
<thead>
<tr>
<th>Group Size (Mean)</th>
<th>Obs.</th>
<th>Mean</th>
<th>NE</th>
<th>St.Dev.</th>
<th>Mean – NE</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 or 6 (4)</td>
<td>201</td>
<td>2.2239</td>
<td>1.0000</td>
<td>2.0651</td>
<td>1.2239</td>
</tr>
<tr>
<td>4 or 8 (6)</td>
<td>165</td>
<td>1.9515</td>
<td>0.5625</td>
<td>2.0025</td>
<td>1.3890</td>
</tr>
<tr>
<td>2 or 12 (7)</td>
<td>147</td>
<td>2.5238</td>
<td>0.8750</td>
<td>2.1655</td>
<td>1.6488</td>
</tr>
<tr>
<td>4 or 12 (8)</td>
<td>207</td>
<td>2.0242</td>
<td>0.5000</td>
<td>2.0490</td>
<td>1.5242</td>
</tr>
</tbody>
</table>

* The average contribution when the lower bound is 2: 2.3506
* The average contribution when the lower bound is 4: 1.9919
* Difference between ‘the lower bound 2’ and ‘the lower bound 4’: 0.3586***
* Difference between ‘2 or 12’ and ‘4 or 8’: 0.5723***

Total 36 subjects participated in the Treatment Group experiment. In each round the uncertainty of the group size is randomly chosen and displayed on the top of the game screen. Difference numbers of observations are due to such randomness. The realization of the group size is informed after all participants made decisions. As the fundamental idea of Proposition (1) suggests, the average contribution level decreases as the lower bound of the group size increases. The difference between the average contribution levels when the lower bound is two and when it is four is statistically significant with the one percent of significance level. Also subjects contributed more when the population is more volatile. The difference between the average contribution when the group size was either two or twelve and when it was either four or eight is statistically significant with the one percent significance level.

**Table 6: Main Experiments**

<table>
<thead>
<tr>
<th>Module</th>
<th>Obs.</th>
<th>Group Size</th>
<th>Mean</th>
<th>NE</th>
<th>St.Dev.</th>
<th>Mean-NE</th>
<th>DO Adjustment(*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>'control'</td>
<td>217</td>
<td>4</td>
<td>1.8018</td>
<td>0.7500</td>
<td>1.8263</td>
<td>1.0518</td>
<td>1.0518</td>
</tr>
<tr>
<td></td>
<td>84</td>
<td>6</td>
<td>1.5952</td>
<td>0.5000</td>
<td>1.4736</td>
<td>1.0952</td>
<td>1.0952</td>
</tr>
<tr>
<td></td>
<td>99</td>
<td>8</td>
<td>1.6465</td>
<td>0.3750</td>
<td>1.5992</td>
<td>1.2715</td>
<td>1.2715</td>
</tr>
<tr>
<td>'treatment'</td>
<td>177</td>
<td>2 or 6</td>
<td>1.8814</td>
<td>1.0000</td>
<td>1.5821</td>
<td>0.8814</td>
<td>1.4175</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>4 or 8</td>
<td>1.7125</td>
<td>0.5625</td>
<td>1.4337</td>
<td>1.1500</td>
<td>1.6861</td>
</tr>
<tr>
<td></td>
<td>143</td>
<td>2 or 12</td>
<td>1.7902</td>
<td>0.8750</td>
<td>1.3767</td>
<td>0.9222</td>
<td>1.4583</td>
</tr>
</tbody>
</table>

* Differences between ‘2 or 6’ and ‘4’: 0.3657**
* Differences between ‘4 or 8’ and ‘6’: 0.5909***

Total 40 subjects (19 in the first session, 21 in the second session) participated in the main experiments. In the first session the ‘control’ module was conducted first, while in the second session the ‘treatment’ was. Each round in the ‘control’ module, the group size was randomly determined and displayed on the top of the game screen. Each round in the ‘treatment’ module, the uncertainty of the group size was randomly chosen and the realization of the group size was announced after all subjects made choices. The contribution levels in the treatment module were higher than those in the corresponding control module.

* The last column shows driving-out-effect-adjusted contribution level. In the baseline experiments the overall contribution level with population uncertainty was at least 0.5361 tokens smaller than that without population uncertainty. After controlling for the Nash equilibrium contribution level, I adjusted the net contribution level by adding 0.5361.
References


**Appendix A**

Sample Experimental Instructions

[Let participants sign in.]

Hi everyone. Welcome to the group-decision experiment. Please make sure you have a pen, two copies of the consent form, and a table which you will need during the experiments. We want you to focus on the experiment, so please do not anything unrelated to the experiment. Please take a moment to turn off your cell phone and any electronic devices with sound notifications. Please read carefully and fill out one copy of the consent form. The form requires your initials at the end of the first page, and your signature at the end of the second page. Note that one copy is for our records, and the other copy is for your records. Please raise your hand if you are done filling out the form.

[Let them fill out the consent form.]

As you have read the consent form, we will conduct experiments in group decision-making. At the end of the experiment you will be paid in cash. The amount of money you earn will depend on the decisions you make, as well as the decisions other participants make.
The whole experiment consists of several, but no more than 30 rounds of simple games whose overall procedure is the same. In each round, you will be asked to make a choice in a slightly different situation and input your decision to the computer interface.

Your choices and answers will be linked with a computer number of your seat. We will never link names with your responses in any way. Your personal information provided for your payments will never be stored nor used for any research. You will neither learn about other participants’ identity in the course of this experiment, nor will they learn about your identity.

You are not allowed to talk to others during the experiments. If you have a question, please raise your hand so that I can come to you and answer your question in private.

Now please wake up your computer. You will see a welcome screen in a yellow window. This is a tutorial which will introduce what you will do during the actual experiments, and make you feel familiar. This tutorial is self-contained, so please read through and follow instructions. I especially recommend you to take a careful look on the table I distributed, because your earnings are mainly determined by the table. I will give you ten minutes to play with the tutorial. If you finish it earlier, you may want to start over. Note that I set computers choose their decision randomly, so computers’ decisions may or may not be similar to that of real participants.

[Give them 10 minutes for the tutorial.]

We are ready to begin. Please close the tutorial. Click on “Experiment” on the taskbar. Everything will be the same with the tutorial, except two things: First, now you play with real participants here and second, your earnings will be paid. The total sum of your earnings will be stored in the server computer, so do not worry about the payments and please focus on the experiment by itself. Let’s start the first round.

[After the experiment ends.]

All experiments are done. Please bring all of your materials to the front one at a time. Another set of experiments will be conducted this week, so please do not share your results with anyone until the end of the week.

[Pay subjects in a debriefing area and dismiss them one at a time.]

Appendix B

Screen Captures of the Tutorial (for Main Experiments)
You have 10 virtual tokens per round. Your task is to allocate the tokens to a private account and a group account in order to earn as much as you can.

Every token in the private account has a value of 10 cents. You also earn money from tokens assigned to the group account according to the table I gave you.

For example, if 11 tokens are gathered in your group account, no matter how much you contributed, you and every group member earn 73 cents from the group account.

So, if you allocate ‘x’ tokens in your private account and your group members allocate 11 tokens in the group account, then ‘73 + 10x’ cents are your earnings for the round.

Your total earnings for the game will be a sum of earnings per round.

You will be reassigned to a new group every round. The group will be formed in two different ways.

In the first module, the group size will be known, and it will be indicated on the top of the game screen.

Module consists of 4 participants. [Please allocate your tokens.]

I allocate ▲ tokens in the group account, and ▼ tokens in the private account.

Total 12 tokens are gathered in the group account. You earned 123 cents for this round.
Tutorial

In the second module, the group size will be either one of two numbers.

For example, if you read “Group size will be either 2 or 6,” your group will consist of 2 or 6 participants with equal(50%) chance. The actual group size will be realized after everyone made a decision.

Got it

Module: 2
Group size will be either 4 or 8.
Round: 6

Please allocate your tokens.

I allocate tokens in the group account, and tokens in the private account.

Reset Submit

Module: 2
Group size will be either 4 or 8.
Round: 6

Group size for this round is realized to 8.
Total 26 tokens are gathered in the group account.
You earned 179 cents for this round.

Click Here to Proceed

Module: 2
Group size will be either 4 or 8.
Round: 6

Thanks for playing!
If you finish the tutorial earlier, you may want to start over.